# Unit I: One-Dimensional Kinematics

A large part of Physics is called mechanics. Mechanics is the study of motion. It is divided into two parts: kinematics and dynamics. Kinematics describes motion, while dynamics looks at the cause of the movement.

## Motion

Motion is the movement of an object from one place to another. It is measured over a time interval and is relative to a reference point. It is often best if the reference point is something stationary such as the ground. An object is in motion if its position is changing with respect to an observer.

### Frames of Reference

Let's say I am standing on the back of a pickup truck that is motionless and I am throwing apples forward. I know that I can throw an apple at exactly 15 m/s every time. If a person were standing on the sidewalk, how fast would she say the apples are moving?

Now the truck starts moving forwards at 20 m/s. I am still throwing apples forwards, exactly the same as I was throwing them before, at 15 m/s. How fast would the spectator say the apples are moving?

How fast according to me does it look like the apples are moving?

When you are standing on the ground, that is your frame of reference. Anything that you see, watch, or measure will be compared to the reference point of the ground. If I am standing in the back of the moving truck, the truck is now my frame of reference and everything will be measured compared to it. We say that moving objects have relative velocity.

Sitting at your desk, how fast are you moving relative to the ground? Relative to the sun? Which answer is correct?

We show different motions as arrows in the direction objects are moving. We call these vectors.

## Vectors

While motion can be described with words, that is often not good enough. Physics is a mathematical science, so we use two categories of mathematical quantities to describe motion:

Scalars

-quantities described with a magnitude (or number) only

-Ex. Temperature, mass, and speed

Vectors

-quantities described with both magnitude and direction

-Ex. Velocity, force, acceleration, displacement, weight, momentum

Ex. Which of these is a scalar? A vector?

8 m 26.4 m/s East 84°C 52.0 m/s 3 km [N]

## Drawing Vectors to Scale

Vectors are used to represent vector quantities on a diagram. A vector is composed of a line segment drawn to scale with an arrowhead at one end. The tail of the vector is at its origin and the head is at the terminal point (the arrowhead). The length of the vector represents its magnitude and the arrowhead indicates its direction. When drawing vectors you must also include reference coordinates.

Ex. Draw 24 km [E] to scale

Ex. Draw 60 m [S27°E].

60 m [S27°E] could also be represent by 60 m [27° E of S] or 60 m [297°]. When using the cardinal directions, your angle should be less than 90°.

Collinear vectors are vectors that exist in the same dimension. In other words, they exist either in the same direction or in the opposite direction. Non-collinear vectors are vectors that exist in more than one dimension (i.e. they are located along different straight lines).

## Adding Vectors Graphically

Add vectors by drawing them head to tail. That is, each vector starts where the last one stopped.

The sum (resultant) is the vector from the tail of the first to the head of the last vector. Measure its length and direction to the same scale.

Ex. 8 m [E] + 3 m [W]

Ex. 4.2 m/s [N37°W] + 2.1 m/s [S15°E]

## Subtracting Vectors Graphically

Subtracting vectors is like subtracting integers – just add the opposite.

Ex. 12 m [W] – 6 m [E]

Ex. 360 km/h [15° E of N] – 270 km/h [120°]

## Mathematical Operations with Vectors

If you multiply a vector by a scalar, you only multiply the magnitude of the vector with the scalar. The direction does not change. The same goes for dividing.

Ex. 2(3.7 m [N23°E])

Ex. 57 N [32°] ÷ 13

Vectors can be added mathematically by the vector component method. The vector is broken down into its x and y components. The collinear components are then added together and the Pythagoras theorem is used to find the magnitude. Trigonometry gets you the angle. Remember SOH CAH TOA!

Ex. Find the x and y components of 150 km [N26°W]

Ex. A football player is hoping to score a touchdown by running a complicated play. He runs 5 yards [N 36° E], then changes his path to run 12 yards [N 52° W] where he meets a block and is forced to run 15 yards [S 73° E] before he is tackled. Determine the resultant vector. Did this player make a successful play?

Step 1: Write down the given information and determine the angle for each vector F, where θ is measured counterclockwise from the positive x-axis [principle angle].

Step 2: Break each vector down into its x and y components by using a trig ratio. SOH CAH TOA is your friend.

Step 3: Add the collinear vectors algebraically.

$$\sum \vec{V}_x = \vec{V}_{x1} + \vec{V}_{x2} + \vec{V}_{x3} + \cdots$$

$$\sum \vec{V}_y = \vec{V}_{y1} + \vec{V}_{y2} + \vec{V}_{y3} + \cdots$$

Step 4: Use the Pythagorean theorem to find the magnitude of the resultant vector.

$$\vec{V}_R = \sqrt{\left(\sum \vec{V}_x\right)^2 + \left(\sum \vec{V}_y\right)^2}$$

Step 5: Use a trigonometric ratio to determine the angle of the resultant vector

Step 6: Convert this angle into cardinal directions when it makes sense.

## Velocity Vectors

A diagram often goes a long ways in solving physics problems. When dealing with velocity vectors, you need to be sure you are properly representing what is happening. The vector diagram used needs to show which vector is "pushing" which vector.

Ex. A man is rowing across a river. He heads north at 12.0 m/s while the river flows east at 4.0 m/s. What is his actual velocity?

Ex. A pilot wants to fly west. His plane has an airspeed of 150 km/h. There is a wind blowing north at 10.0 km/h. What is the groundspeed and proper heading?

## Motion

Uniform motion continues in a straight line path at a constant speed. Non-uniform motion is either not in a straight line or not at a constant speed.

Recall that average speed is equal to the distance travelled divided by the time it took.

| | |
|---|---|
| $v_{av} = \frac{\Delta d}{\Delta t} = \frac{d_2 - d_1}{t_2 - t_1}$ | Where v = average speed<br>d1 = initial position<br>d2 = final position<br>Δt = total time taken to travel the distance |

Ex. If it took you 2.3 hours to travel the 258.4 km to Saskatoon, at what speed were you travelling?

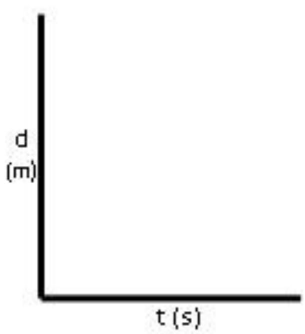
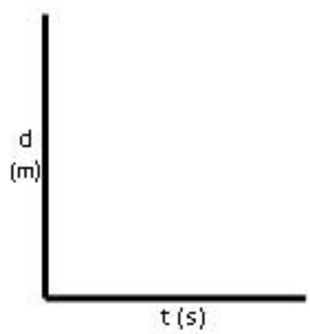
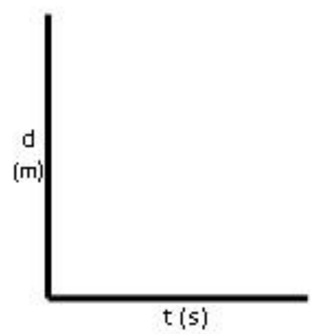
# Graphing Motion

## Distance-Time Graphs (Position-Time Graphs)

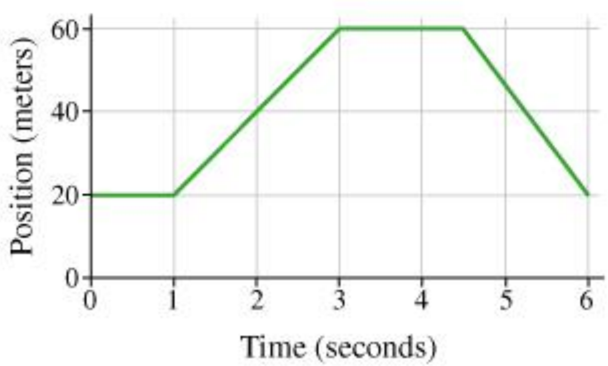
Distance needs no reference frame. You measure the distance between two objects by measuring their separation. It simply refers to the length (or magnitude) between the two objects. Direction does not matter.

The position of an object refers to the change of an objects original starting point to its ending point. In this case, magnitude and direction of motion are important. This means it is a vector unit.

Constant (uniform) motion is given by a straight line on the graph. Curves indicate non-uniform motion. Zero slope represents no motion.

| No speed | Constant Speed | Changing Speed (Acceleration) |
|---|---|---|
|   |   |   |

Ex.

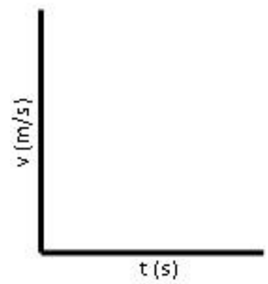
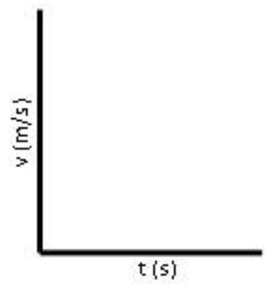
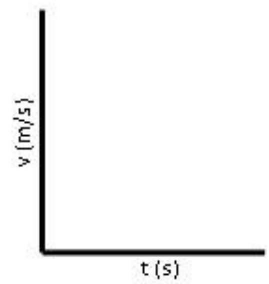
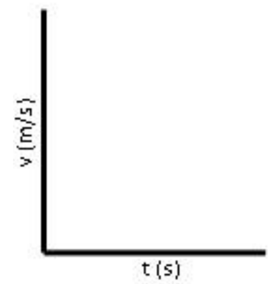



A) Find the average speed from 1 to 3 s.

B) What total distance did the object travel?

## Speed–Time Graphs

Uniform motion is a zero slope line.  A non-zero slope shows constant acceleration.

| No speed | Constant Speed | Acceleration | Deceleration |
|---|---|---|---|
|   |   |   |   |

Deceleration is just a negative acceleration, which would be acceleration in the opposite direction.

## Displacement and Velocity Graphs

Displacement refers to an objects change in position from its starting point.  It is a vector unit, so the direction is important.  To find displacement on a velocity-time graph, find the area under the curve.

Velocity is an object's displacement divided by the period of time the displacement occurred in.  It is a vector quantity.

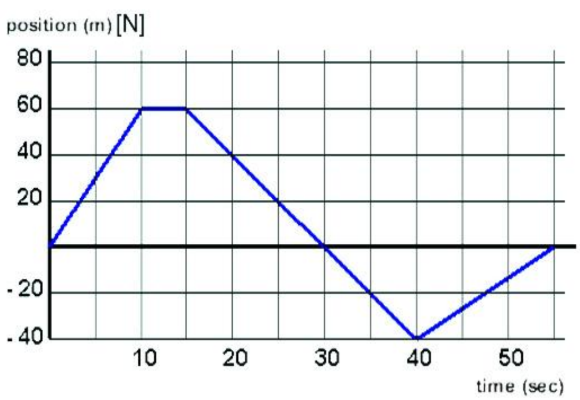
$$\vec{V}_{av} = \frac{\Delta\vec{d}}{\Delta t}$$

Where $\Delta\vec{d} = \vec{d}_2 - \vec{d}_1$

$\Delta t = t_2 - t_1$

$\vec{v}_{av}$ = average velocity

Ex.



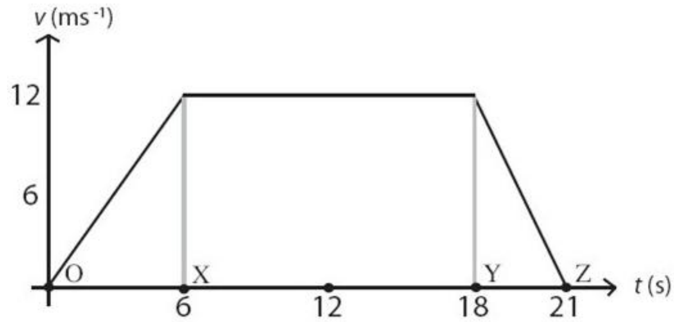

a) Find the distance travelled from 10 s to 40 s.

b) Find the displacement from 10 s to 40 s.

c) Find the average speed from 10 s to 40 s.

d) Find the average velocity from 10 s to 40 s.

Ex. A moped's velocity, as it is travelling between 2 traffic lights is shown below. Assume a [forward] direction.




a) When was the moped going a uniform velocity?

b) When was the moped accelerating? How can you tell?

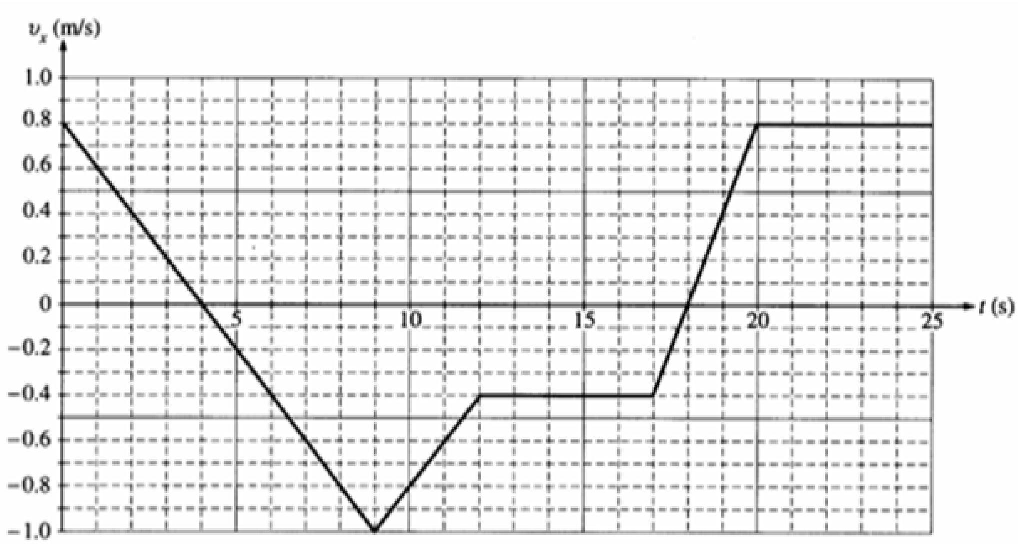
c) When was the moped decelerating? How can you tell?

d) How fast was the moped going at 10 s?

e) Calculate the acceleration of the moped between 18 s and 21 s.

f) What was the displacement of the moped between 0 s and 21 s? (This is the total distance between the traffic lights)

Ex. A leaf blowing in a forward direction. Calculate the displacement from 0 s to 20 s.



# Acceleration

Acceleration is the rate of change of velocity based on a unit of time. Remember, if acceleration is negative, then the object is decelerating; it is slowing down. If the acceleration is zero, then the object is moving at a constant velocity or may not be moving at all. Since acceleration is a vector, simply changing your direction but not your speed is also a form of acceleration.

| | |
|---|---|
| $\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t}$ | where $\vec{a}_{av}$ = average acceleration<br>$\Delta\vec{v}$ = change in velocity<br>$\Delta t$ = change in time |

Ex. The velocity of a car increases from 2.0 m/s at 1.0 s to 16 m/s at 4.5 s. What is the car's average acceleration?

Ex. A car goes faster and faster backwards down a long driveway. The car's velocity changes from -2.0 m/s to -9.0 m/s in a 2.0 s time interval. Find its acceleration.

# Acceleration and Escape Velocity

See handout.

# Equations of Motion

## Velocity of An Object With Constant Acceleration

Acceleration that does not change in time is uniform or constant acceleration. The slope of the line on a velocity-time graph gives the acceleration. Finding the equation of this line, or just rearranging our average acceleration formula gives:

| | |
|---|---|
| $\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t$ | where $\vec{v}_2$ = final velocity<br>$\vec{v}_1$ = initial velocity<br>$\vec{a}$ = acceleration<br>$\Delta t$ = time interval |

Ex. If a car with a velocity of 2.0 m/s at t = 0 accelerates at a rate of 4.0 $m/s^2$ for 2.5s, what is its velocity at t =2.5 s?

Ex. A car is travelling with a velocity of 1.23 m/s when t = 5.00 s. It has an unknown constant acceleration, but when t = 10.0 s, it has a velocity of 24.50 m/s. What is the car's acceleration?

## Displacement When Velocity and Time Are Known

To find the displacement if the object is uniformly accelerating, the velocity is replaced by the average velocity.

$$\boxed{\vec{v}_{av} = \frac{\vec{v}_2 + \vec{v}_1}{2}}$$

Using a velocity-time graph, we can find a formula for the area under the curve to give us displacement:

$$\boxed{\Delta\vec{d} = \frac{1}{2}(\vec{v}_2 + \vec{v}_1)\Delta t}$$

This equation states that displacement is equal to average velocity times time.

Ex. What is the displacement of a train as it is accelerated uniformly from 11 m/s to 33 m/s in a 20.0 s interval?

Ex. Find the initial velocity of the train if the final velocity is 96 km/h and it covered a distance of 730 km in 9.0 h.

## Displacement When Acceleration and Time are Known

By combining two of the above equations, we get the following:

$$\Delta\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}(\Delta t)^2$$

If acceleration is zero, this equation simply turns into d = vt.

We can also get:

$$\Delta\vec{d} = \vec{v}_2\Delta t - \frac{1}{2}\vec{a}(\Delta t)^2$$

Ex. A car starting from rest accelerates uniformly at 6.1 m/s$^2$ for 7.0 s. How far does the car move?

Ex. Find the acceleration of an object when the initial velocity is 12 m/s, the time is 7.0 s, and the displacement is 382 m.

## Displacement When Velocity and Acceleration Are Known

Again, combining equations from above gives us another equation:

$$\vec{v}_2^2 = \vec{v}_1^2 + 2\vec{a}\Delta\vec{d}$$

This is useful when you don't know the time interval the motion occurred over.

Ex. Find the final velocity of an object if the initial velocity is 17 m/s, the acceleration is 3.4 m/s$^2$, and the displacement is 1400 m.

Ex. An airplane must reach a velocity of 71 m/s for takeoff. If the runway is 1.0 km long, what must the constant acceleration be?

Ex. Jack and Jill ran down the hill. Both started from rest and accelerated steadily. Jack accelerated at $0.25 m/s^2$ and Jill at $0.30 m/s^2$. After running 20.0 s, Jill fell down.

a) How far did Jill get before she fell?

b) How far had Jack travelled when Jill fell?

c) How fast was Jack running when Jill fell?

d) How long was it after Jill fell that Jack ran into her and broke his crown (to the nearest second)?

# Unit II: Free-fall and Projectile Motion

## Aristotle

From the time of Aristotle (384-322 BC) until the late 1500's, gravity was believed to act differently on different objects.

- This was based on Aristotle's observations of doing things like dropping a metal bar and a feather at the same time. Which one hits the ground first?
    - Obviously, common sense will tell you that the bar will hit first, while the feather slowly flutters to the ground.
- In Aristotle's opinion, this was because the bar was being pulled harder (and faster) by gravity because of its physical property of having more mass.
- Because everyone could see this when they dropped different objects, it wasn't questioned for almost 2000 years.

## Galileo

Galileo Galilei was the first major scientist to refute (prove wrong) Aristotle's theories.

- In his famous (at least to Physicists!) experiment, Galileo *supposedly* went to the top of the leaning tower of Pisa and dropped a wooden ball and a lead ball at the same time.
    - Both were the same size, but different masses.
- Down below an assistant watched for them to hit the ground.
    - They both hit the ground at the same time, even though Aristotle would say that the heavier metal ball should hit first.

Galileo had shown that the different rates at which some objects fall is due to air resistance, a type of friction.

- Get rid of friction (air resistance) and all objects will fall at the same rate.
- Galileo said that the acceleration of any object (in the absence of air resistance) is the same.

To this day we follow the model that Galileo created.

In the absence of air resistance, all objects fall at a constant acceleration, regardless of their mass.

This acceleration is called the acceleration due to gravity.

$$\boxed{Acceleration\ Due\ to\ Gravity = \left|\vec{a}_g\right| = g = 9.81\ m/s^2}$$

In any of the questions we will do, we must assume that the entire problem happens near the surface of the Earth. We will learn in a later unit that the acceleration due to gravity does decrease as you move further away from the centre of the earth. The magnitude of g also changes on different celestial bodies.

Since gravity is just an acceleration like any other, it can be used in any of the formulas that we have used so far. Just be careful about using the correct sign (positive or negative) for the variables in the problem. I strongly suggest that you stick with up being positive and down being negative.

| | |
|---|---|
| Upwards<br>+ velocity<br>+ displacement | Downwards<br>- velocity<br>-displacement<br>- gravity |

Ex. A ball is thrown up into the air with an initial velocity of 26.3 m/s. Determine its velocity after 1.80 s have passed.

Ex. I throw a ball down from the top of a cliff so that it leaves my hand moving at 12 m/s. Detemine how fast it is going 3.47 s later.

Ex. A jackalope jumps up into the air. As he leaves the ground he is travelling at 16.0 m/s. How fast is he going after 2.8 s?

# Throwing an Object Up Into the Air

There are a few rules that you have to keep track of. Let's look at the way an object thrown up into the air moves.

## Going up…

1) It starts at the bottom at the maximum speed.
2) As it rises, it slows down because gravity is a negative acceleration.
3) It reaches its maximum height, where for a moment its instantaneous velocity is zero. This is exactly half ways through the flight time if it is landing at the same height it was thrown.

## Coming down…

1) The ball begins to speed up, but downwards. Because gravity is negative, the velocity of the object increases in the negative direction.
2) When it reaches the same height that it started from (like the ground, or the person's hand), it will be going at the same speed down as it was originally moving up at. The only difference is that this velocity is negative because it is pointing down.
3) It takes just as much time to come down as it did to go up if it is landing at the same height it was thrown.

Applying these rules might seem complicated, but since they stay the same all the time you can get used to the problems by just practicing them over and over again.

Ex. I throw my ball up (again) at a velocity of 12 m/s.

a) Determine how much time it takes to reach its maximum height.

b) Find how high the ball goes.

c) Determine how fast it is going when it reaches my hand again.

## Gee's

You might have heard people flying fighter jets or rockets in movies say how many "gee's" they were feeling. All this means is that they are comparing the acceleration they are feeling to regular gravity.

So, right now just sitting in a chair, you are experiencing 1 gee - regular gravity. This means that you are experiencing one times the acceleration of gravity. One times 9.80 $m/s^2$ is equal to 9.80 $m/s^2$.

During lift-off the astronauts in the space shuttle experience about 4 gee's. That works out to about 4 x 9.80 = 39 $m/s^2$.

Ex. Acceleration due to gravity on the moon is 1.67 $m/s^2$. Determine how many gee's this is.

Ex. A space probe sent to one of Jupiter's moons, Callisto, is taking pictures with a digital camera that records one picture every tenth of a second (measured to a precision of three sig digs). While doing this, a small screw falls off a part of the probe right in front of the camera and can be seen to start falling past the camera as it takes the pictures. Over a series of six pictures, the screw can be seen to fall 15.7 cm starting from rest. Determine how many gee's there are on this moon.

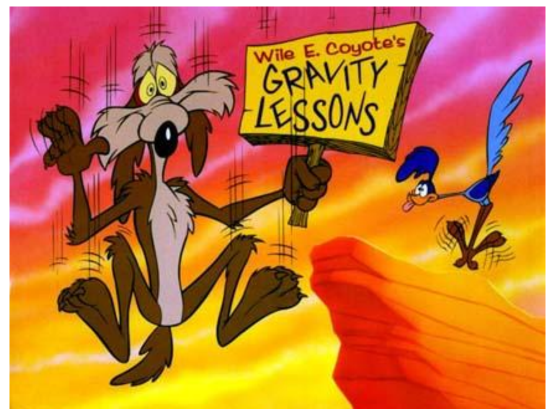
## Projectiles Launched Horizontally

A projectile is any object which is thrown or otherwise projected into the air. Again, we assume that the effects of air resistance are negligible for the problems we deal with.

The study of projectile motion brings together a lot of what you have learned in the past few sections. You need to know about gravity, velocity, acceleration, and vector components to be able to fully understand (and figure out) these questions. Don't worry, though. Even with all that stuff to keep track of, learning how to do these questions and understand what is happening a little bit at a time makes it all manageable.

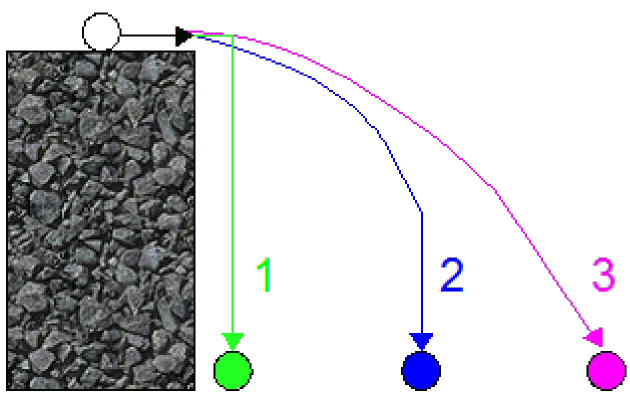
### Wile E. Coyote

I'm sure you have seen the cartoon where the Coyote is chasing after the Road Runner and runs off of the cliff. He hangs in midair for a second, looks down, and then starts to fall. The question is how true is this, and how many people believe it is true?




A few years ago some researchers in the U.S. went to elementary schools, junior and senior high schools, and universities and asked them to look at the following:

*"Ignoring air resistance, which of the following correctly shows what an object would do if it rolled off a cliff?"*




The correct answer is actually number 3, and if you think about it using the physics you've studied it makes sense.

Let's say a coyote does run off a cliff. As he leaves the cliff he has a horizontal velocity. From studying forces and acceleration you already know that the only way to change that horizontal velocity (cause a horizontal acceleration) is to exert a horizontal force on the coyote.

If we are ignoring air resistance (which is a very good idea since it will be practically zero), then there is no horizontal force to cause a horizontal acceleration. Since there is no horizontal acceleration, the coyote will travel horizontally at the same speed the whole time!

That doesn't tell us anything about what is happening vertically, which is completely separate from what the object is doing horizontally.

As soon as the coyote leaves the cliff he will experience a vertical force due to gravity. This force will cause him to start to accelerate in the vertical direction. As he falls he will be going faster and faster in the vertical direction.

Looking at this problem as what is happening horizontally and vertically, you should get the idea that this is just like the components of vectors that we were just working on a couple of lessons back.

The horizontal and vertical components of the motion of an object going off a cliff are separate from each other, and cannot affect each other. In a lot of books you will see the horizontal component called x and the vertical component called y.

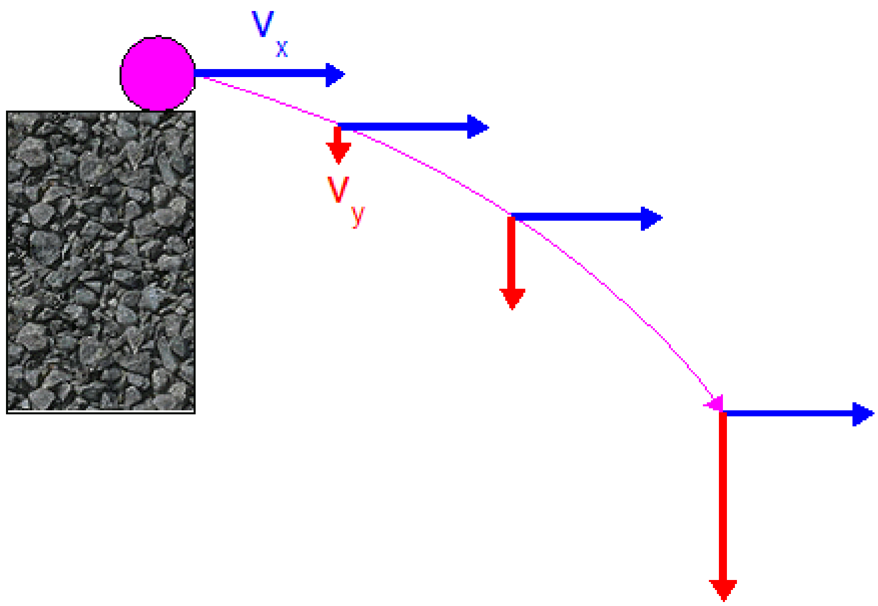



*Illustration 2: Components as a ball rolls off a cliff.*

The x-component is there from the start, and stays the same the entire time.

The y-component doesn't even exist at the beginning, but grows bigger as the object falls.

The shape of the path that it follows is actually a parabola. If you've studied those in math class, great! Don't worry about it if you haven't, just as long as you recognize the shape and know the name.

To understand how to actually figure out questions involving these situations, it's probably best to look at an example. Keep in mind the characteristics of the object as it falls while you go through the example. When you are doing a part of a question that has to do with vertical movement "THINK VERTICAL" and only use vertical ideas (like gravity). When you are doing a part of a question that has to do with horizontal movement "THINK HORIZONTAL" and only use horizontal ideas (no gravity/acceleration).

Ex. I throw a ball off the edge of a 15.0m tall cliff. I threw it horizontally at 8.0 m/s.

a) Determine how much time it takes to fall.

b) Determine how far from the base of the cliff it hits the ground.

c) Determine how fast it is moving vertically when it hits the ground.

d) Determine what its total velocity is when it hits the ground.

Although there will always be slight differences in actual problems, this is the standard sort of question that you will be asked for these types of problems.

Ex.  A stuntman is to run across a rooftop and then jump over a 5.00 m gap landing on the roof of the next building.  The building he is jumping from is 5.0 m higher than the next building.  If the stuntman is running at 4.80 m/s will he make it?

Summary:

- Horizontally, a projectile travels at a constant velocity.
- Vertically, a projectile travels at the same rate as an object experiencing free-fall motion.
- To analyze projectile motion it is useful to consider the vertical (y) and horizontal (x) components of the velocity and displacement separately.
- The horizontal motion of a projectile depends on the horizontal component of the initial velocity $(\vec{v}_x)$.
- The horizontal displacement $(\vec{d}_x)$ of a projectile, relative to the ground, can be found using the horizontal component of the velocity and the total flight time $(\Delta t)$.  This displacement is called range.
- The vertical motion of a projectile depends on the vertical component of the initial velocity and on the acceleration due to gravity.
- The vertical motion of a projectile can be determined with the equations for free-fall motion using vertical components and the acceleration due to gravity.

## Projectile Motion at an Angle

To do questions involving objects launched from the ground upwards at an angle (like kicking a football up into the air and watching it as it arcs in the air and comes back down), you need to add a few more steps to the way you did the questions for objects launched horizontally.

There are actually two ways to do these types of problems, one based on the vertical velocity of the object, the other based on the vertical displacement.

The only big difference in these methods is how we are going to calculate the time that the object spends in the air.  Choose whichever method you are most comfortable with, and whichever one suits the particular question you are doing.  We will look at each and then it's up to you to figure out which way you will approach a problem.

Imagine for a moment that you are watching an object as it rises into the air after you kick it upwards at an angle. Look at Illustration 1 below as you read through this description.

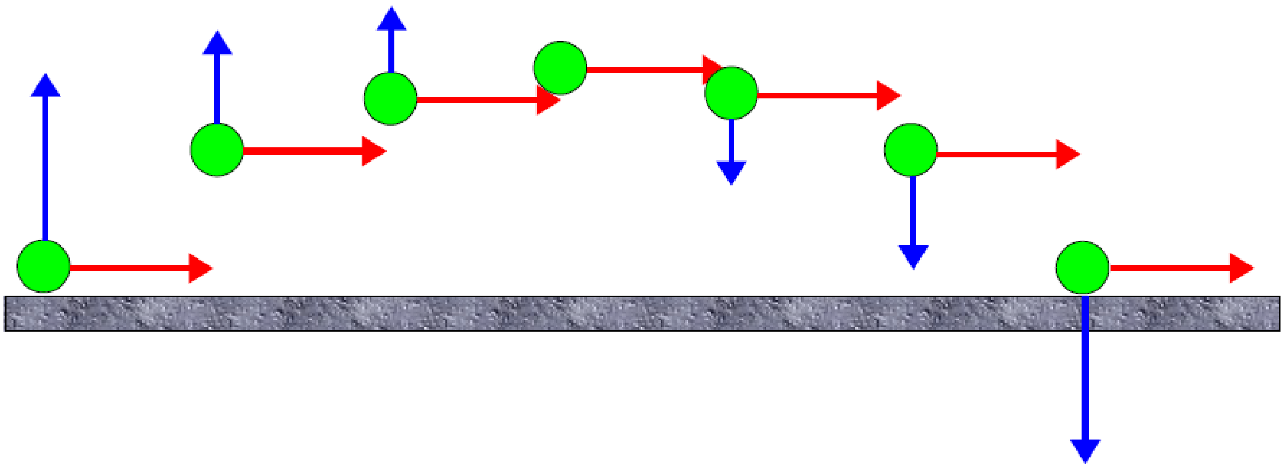


*Illustration 1: Object launched at an angle.*

When it left your foot, it was going at the fastest that it can possibly move during its flight. The instant it leaves your foot, gravity is pulling down on it, causing it to have less and less vertical velocity. Remember that there will be no change in the horizontal component of its velocity.

When it reaches the highest point in its flight, it isn't moving up, and it isn't moving down, for an instant of time. Its vertical velocity is zero!

By the time it reaches the ground again, it will still be moving with its original horizontal velocity and will have just as much vertical velocity as when it left your foot. It will have the exact same velocity as it left your foot with.

We can use this information about its vertical movement to do some calculations. We know:

- that there is gravity ($-9.80 m/s^2$) causing the acceleration on the object vertically.
- the initial vertical velocity of the object.
- the final vertical velocity of the object. We can even use this two ways, since we can say that the final vertical velocity happens at the halfway point (zero m/s), or when it gets back to the ground (same as it left the ground at).

This gives us enough information to calculate the maximum height of the flight, and the time it spends in the air. After that, we can calculate just about anything.

Ex. You kick a soccer ball at an angle of 40° above the ground with a velocity of 20m/s. Determine:

a) How high will it go?

b) How much time does it spend in the air?

c) How far away from you will it hit the ground (aka *range*)?

d) What is the ball's velocity when it hits the ground?

Ex. A basketball player tries to make a half-court jump shot, releasing the ball at the same height as the basket. Assuming the ball is launched 14 m from the basket at 51°, find the velocity the player must shoot the ball.

# Unit III: Dynamics

Dynamics is the study of why objects move – it is the study of forces. Kinematics and Dynamics together form what is called Mechanics. Two people were instrumental in the birth of this branch of physics:

## Galileo Galilei

- Born in Italy in 1564 and died in 1642
- Remembered for his work in astronomy, mathematics, and physics
- Made the Catholic Church grumpy because he challenged Aristotelian notions about motion by performing experiments
- In 1633 the Inquisition forced him to renounce his theories and placed him under house arrest

## Sir Isaac Newton

- Born in England in 1642 and died in 1727
- Invented Calculus
- Came up with three Laws of Motion
- Not friendly, but always acknowledged the work of those before him, especially Galileo:
    - "If I have seen further than other men, it is because I have stood on the shoulders of giants."

## Force

A force is push or a pull. Since these can have different magnitudes and directions, force is a vector quantity. Any force acting on an object can change its shape, its velocity, or both.

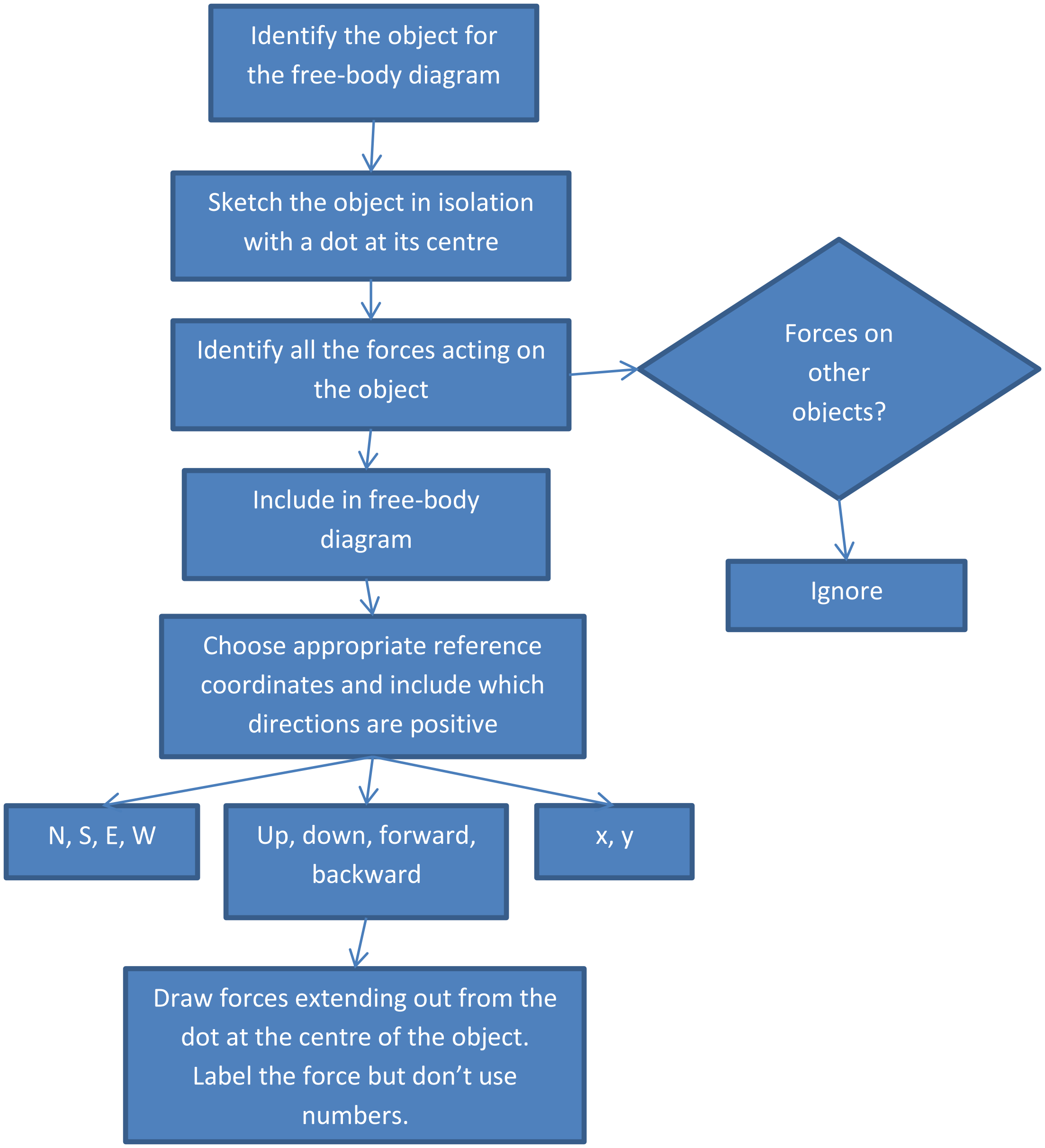
The symbol for force is $\vec{F}$ and the unit is the Newton (N). One Newton is equal to 1 kgm/s$^2$. Direction is denoted with whatever system is most convenient for the situation.

## Free-Body Diagrams

Often, there are many forces acting on an object in any given situation and a system is needed to organize these forces. We will use something called a free-body diagram. A free-body diagram is a sketch that shows the object all by itself, isolated from all other objects.

When drawing a free-body diagram, it is important to denote your reference coordinates that are being used.

Flowchart for drawing free-body diagrams:




Ex. Sketch free-body diagram for a ball sitting on your palm.

We will see many types of forces acting on an object.  Some common that you need to know include:

$$\vec{F_g} = force\ due\ to\ gravity$$

$$\vec{F_a} = applied\ force$$

$$\vec{F_f} = force\ due\ to\ friction$$

$$\vec{F_T} = force\ of\ tension$$

$$\vec{F_N} = normal\ force$$

$$\vec{F}_{NET} = net\ force$$

Each of the above forces will be discussed as they are introduced.  When labeling your diagrams, you need to use the same symbols shown above.  Capitals and lower case letters do matter.

Ex.  Sketch a free-body diagram of a textbook sitting on a table.

The normal force $(\vec{F_N})$ is force on an object that is perpendicular to the contact surface.

Ex.  You are pushing a sofa across a rough floor.  Sketch the free-body diagram of the sofa.

The force due to friction $(\vec{F_f})$ always opposes the motion of an object and is parallel to the contact surface.

Ex.  A tobogganer is sliding down a hill.  Sketch a free-body diagram of the toboggan.

## Net Force

The net force acting on an object is the resultant of all forces acting on an object. Equilibrium exists if the net force is zero. We find it by finding the sum of all the forces acting on the object.

$$\vec{F}_{NET} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots$$

An object in equilibrium will move with a constant velocity.

An unbalanced force exists when the resultant of all the forces acting on an object does not equal zero. If no external unbalanced force acts on an object, its velocity will remain constant. If an unbalanced force exists, the object will begin to accelerate in that direction.

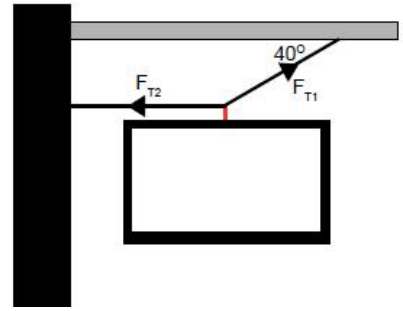
If the forces are parallel to one another, we can just add them together as positive and negative forces. If they are at an angle to one another, we have to add them as components of vectors.

The unit of force is called the Newton in honour of Sir Isaac Newton. It is equivalent to the kg m/s$^2$.

Ex. Mr. Birrell's Porsche is stuck in a snow drift. Two enterprising students attach two ropes to the Porsche and attempt to pull it out by pulling in the same direction. Mason pulls with a force of 72 N while Caden pulls with a force of 85 N. There is a force due to friction of 55N acting on the car. Sketch a free-body diagram and determine the net force acting on the car.

Ex. As Mason and Caden bravely pull the car, they notice a patch of ice on the road directly in front of them. To keep on pulling without wiping out on the ice, they must begin to pull at an angle as they walk around the ice. Mason now pulls with 72 N [E15°N] and Caden pulls with 85 N [E20°S]. The force due to friction is still 55n. Sketch a new free-body diagram and determine the new net force.

Ex. A 20kg sign is supposed to hang from a pair of wires attached to the wall and a support beam as shown in the diagram. The wires that will be used can withstand a force of tension up to 300 N each. Determine the tension in wire one and wire two and explain any concerns you may have.




The force due to gravity is also called weight. It is defined as:

| | |
|---|---|
| $\vec{F}_g = m\vec{g}$ | Where $\vec{F}_f$ = Weight (Newtons, N, kg m/s$^2$)<br>m = mass (kg)<br>$\vec{g}$ = acceleration due to gravity (m/s$^2$) |

## Newton's First Law (Inertia)

**"Every body continues in a state of rest or uniform velocity in a straight line, unless an external force acts on it."**

"Every body…"

"…continues in a state of rest or uniform velocity in a straight line…"

“…unless an external force acts on it.”

Newton called the idea of an object resisting its state of motion inertia. This is why the First Law is sometimes called the law of inertia. We don’t necessarily see it in daily life due to the effects of friction. The first law basically says:

$$If\ \vec{F}_{NET} = 0\ then\ \Delta\vec{v} = 0$$

Ex. Describe the motion of a hockey puck shot down the ice.

Ex. Describe the motion of your binder sitting on your desk.

Ex. Use Newton’s First Law to explain why people are injured in head-on collisions in car accidents when they are not wearing their seatbelts.

Newton essentially nicked his First Law from Galileo. Newton gets credit for it because he was the first to actually formally publish it and back it up with mathematical proofs.

## Newton’s Second Law (Motion)

The First Law says what will happen if there is no net force. The Second Law says what will happen when there is a net force present.

“When an external, unbalance force acts on an object, the object will accelerate in the same direction as the force. The acceleration varies directly as the force, and inversely as the mass.”

Break it down:

“When an external, unbalanced force…”

“…accelerates in the same direction as the force…”

“The acceleration varies directly as the force…”

"…and inversely as the mass."

From the above, we get the following mathematical relationship to express Newton's Second Law:

| | |
|---|---|
| $\vec{F}_{NET} = m\vec{a}$ | Where $\vec{F}_{NET}$= Net Force (Newtons, N, kg m/s$^2$)<br>m = mass (kg)<br>$\vec{a}$ = acceleration (m/s$^2$) |

Ex. What is the net force acting on a 8.35 kg object if it is accelerated at 24 m/s$^2$?

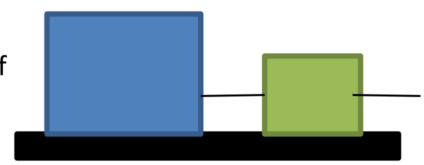
Ex. A lacrosse player exerts an average net horizontal force of 2.8 N [forward] on a 0.14 kg lacrosse ball while running with it in the net of his stick. Calculate the average horizontal acceleration of the ball while in contact with the lacrosse net.

Ex. A person and an elevator have a combined mass of 532 kg. The elevator cable exerts a force of 625 N [up] on the elevator. Find the acceleration of the person.

Ex. Determine the acceleration of the elevator in the example above if the tension in the cable is only 320 N [up].

Ex. A 0.0500 g piece of paper is dropped. While it falls there is a frictional force of $4.71 \times 10^{-4}$ N. Determine the acceleration of the paper.

Ex. A 31kg box and a 7.0 kg box are attached by a thin wire as shown in the picture. A person pulls on the wire attached to the box on the right. There is a force of friction of 243 N acting on the boxes. Find the force applied to the boxes if they accelerate to the right at 3.78 $m/s^2$.



Pulley's change the direction of force. In any pulley problem we do, assume the rope and pulley are massless, have negligible width, and that the rope does not stretch or break. Often, one of the masses will be heavier than the other. This means the heavy mass will move down and the lighter mass move up.

Ex. Two masses are hanging from a pulley. If one mass is 15.25 kg and the mass two is 9.55 kg, determine the acceleration of each mass.

Remember, a pulley can change the direction a force is acting in, but it does not change the magnitude of the force.

## Newton's Third Law (Action-Reaction)

The third law deals with what happens when objects interact.

"For every action force there is an equal but opposite reaction force."

This means that when something applies a force, there will be an equal and opposite force back in the opposite direction. When you push on the handle of a lawnmower to make it go forward, it will push back against you in the opposite direction with just as much force. This is what you feel against your hand. It is important to remember that the two forces you are dealing with are each acting on different objects.

Contact forces are examples of action-reaction forces.

Ex.

Action: the tires on a car push the road

Reaction:

Action: In the lake, you push the water backwards:

Reaction:

Action: A rocket pushes out exhaust

Reaction:

Action: the earth pulls down on an apple

Reaction:

Ex. If the apple has a mass of 0.150 kg determine the acceleration of the Earth. ($M_e = 5.97 \times 10^{24}$)

Ex. When a rifle fires a bullet, the force the rifle exerts on the bullet is exactly the same (but in the opposite direction) as the force the bullet exerts on the rifle. This is what is commonly called "kick-back". If the bullet has a mass of 15 g and the rifle is 6.0 kg, the bullet leaves the 75 cm long rifle barrel moving at 70.0 m/s.

a) Find the acceleration of the bullet.

b) Determine the force of the rifle on the bullet.

c) Determine the acceleration of the rifle.

d) Explain why the bullet accelerates more than the rifle if the forces are the same.

## Friction

Friction acts to oppose motion. To find the direction of the force, look at the direction the object is travelling. The force due to friction acts 180° opposite.

It is not really known why friction acts the way it does. Some people believe it is the tiny imperfections in the two surfaces rubbing against each other. Others believe there are small electrostatic attractions between atoms of the two surfaces pulling on each other. Regardless of how friction acts on the microscopic scale, we can still predict what happens on the macroscopic scale.

Static friction is the friction that exists between two surfaces that are not moving relative to each other. Kinetic (sliding)friction is friction that exists between two surfaces that are moving relative to each other. In most cases, the static friction is greater than the kinetic friction.

If you push on a stationary object, you have to overcome the static friction. Once it is moving, it is often easier because you only have to overcome the smaller kinetic friction.

| | |
|---|---|
| $F_f = \mu F_N$ | Where $F_f$=Force due to friction<br>$F_N$=Normal force<br>$\mu$=coefficient of friction |

The coefficient of friction, $\mu$, is different between any two unique surfaces. It has no unit and is an empirical discovery. This means the value of $\mu$ must be found by experimenting – there is no shortcut formula.

Each pair of surfaces also has two value of $\mu$. The coefficient of static friction is $\mu_s$ and the coefficient of kinetic friction is $\mu_k$. The smaller the value of $\mu$, the less the force of friction there is between the two surfaces.

The static friction that you calculate is a measure of the maximum it can be. It can be any value between 0 and that maximum amount. The kinetic friction you calculate is the value of the friction – it is not a range.

| | |
|---|---|
| $F_{fs} \leq \mu_s F_n$ | $F_{fk} = \mu_k F_n$ |

Ex. A 11 kg piece of wood is sitting on top of another piece of wood. There is 43 N of maximum static friction between them. Find the coefficient of static friction between the two pieces of wood.

As long as the surface is completely horizontal, we can say that $F_N = -F_g = -mg$.

Ex. A 15 kg steel box is sitting on a steel workbench. Wanting to assemble your new Malibu Stacy Dream Mansion, you try to push the box out of the way.

a) Sketch a free body diagram of the box.

b) You try to push against the box with a force of 25 N. Determine if anything will happen.

c) Determine what happens when you push with a force of 73 N.

d) In anger, you Hulk out and push with a force of 120 N. What happens?

By now, you have probably come to hate angles. This is a natural and healthy response, but one that must be overcome. In life, one rarely ever pushes or pulls at a perfectly horizontal angle. Life is not neat; it is a messy affair.

Ex. A person is pushing down on a 63 kg box so that the applied force is 1024 N at an angle of 20° above the horizontal. The coefficient of kinetic friction is 0.42. Determine the acceleration of the box.

## Inclined Planes

Running down an incline plane is easier than running horizontal. Downhill is faster. However, completing an incline plane problem in physics is neither quicker nor easier than a horizontal problem.

Ex. Determine the acceleration of a 15 kg box down a 30° slope if the coefficient of friction is 0.13 on this surface.

FBD:

Pick a convenient coordinate system:

Find $F_g$ and break it down into parallel to the slope ($F_{\parallel}$) and perpendicular to the slope ($F_{\perp}$). $F_{\perp}$ will be equal to the normal force.

Find the force due to friction:

Find the net force:

Find the acceleration of the box:

Go over the example forty-two more times so that you can familiarize yourself with how to break the problem down into manageable chunks. Once you understand each part, it's all downhill from there...

# Unit IV More Applications of Kinematics and Dynamics

## Things to Remember

Period: The time it takes to go through one revolution or cycle. Measured in seconds.

$$T = \frac{t}{cycles}$$

Frequency: How often something repeats itself; the number of cycles per second. Measured in Hz (/s).

$$f = \frac{cycles}{t}$$

Do remember that T and f are inverse of each other:

$$T = \frac{1}{f} \text{ and } f = \frac{1}{T}$$

Ex. A conventional, non-SSD hard drive of a computer is a metal disc that spins while data is written or read from it. The specs listed for a computer you are looking at is that the hard drive spins at 7200 rpm.

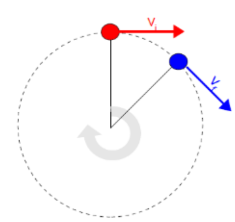
a) What does this mean?

b) Find its frequency and explain this measurement.

c) Find its period.

## Circular Motion

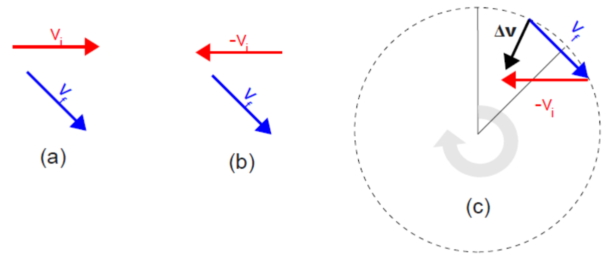
Think of spinning a yo-yo over your head. How would you describe its motion? Clearly, it moves in the path of a circle.



For all of our questions we do, assume that the object travelling in the circle is moving at a constant speed. Do not confuse speed with velocity, as the object is still accelerating due to a change in direction as it moves around the circle.

The velocity vector points along a tangent to the circle, in the direction that the object would tend to move if it were suddenly released.

The acceleration acts in the same direction as the change in velocity. The picture below shows the original vectors from the picture above (a) changed to show $-v_i$ (b) in order to find Δv (c). Notice the direction of Δv will always point towards the centre of the circle.




While travelling in the path of a circle, the acceleration is called centripetal acceleration. It always acts inward, toward the centre of the circle, in the same direction as the change in velocity, perpendicular to the velocity vector. Centripetal comes from Greek meaning “centre-seeking”.

Since we are moving at a constant speed, we can modify $\vec{v} = \frac{\Delta d}{\Delta t}$ to get:

$$\vec{v} = \frac{2\pi r}{t}$$

| | |
|---|---|
| $\vec{v} = \frac{2\pi r}{t}$ | Where v = velocity<br>r = radius of circle<br>t = time for one revolution |

Since t would be the time to go around one complete circle, you could substitute in period:

| | |
|---|---|
| $\vec{v} = \frac{2\pi r}{T}$ | Where v = velocity<br>r = radius of circle<br>T = Period |

## Centripetal Acceleration

The magnitude of the centripetal acceleration is given by:

| | |
|---|---|
| $\vec{a} = \frac{\vec{v}^2}{r}$ | Where a = acceleration towards centre of circle<br>r = radius of circle<br>v = velocity of object |
| $\vec{a} = \frac{4\pi^2 r}{T^2}$ | T = period of revolution |
| $\vec{a} = 4\pi^2 r f^2$ | f = frequency of revolution |

Ex. A DVD disc has a diameter of 12.0 cm and a rotational period of 0.100 s. Determine the centripetal acceleration at the outer edge of the disc

## Centripetal Force

Since we have acceleration when going around a circle, we must also have an accompanying force. Using Newton's Second Law, we can derive a formula for this force that is in the same direction as the acceleration:

| | |
|---|---|
| $\vec{F}_c = m\vec{a}_c$ | Where $F_c$ = centripetal force<br>m = mass of object<br>$a_c$ = centripetal acceleration |

Furthermore, we can substitute any of our formulas for centripetal acceleration into Newton's Second law to get the following:

| | | |
|---|---|---|
| $\vec{F}_c = m\frac{\vec{v}^2}{r}$ | $\vec{F}_c = \frac{4\pi^2 mr}{T^2}$ | $\vec{F}_c = 4\pi^2 mrf^2$ |

Ex. Determine the magnitude of the centripetal force exerted by the rim of a dragster's wheel on a 45.0-kg tire. The tire has a 0.480-m radius and is rotating at a speed of 30.0 m/s

Ex. Determine the maximum speed with which a 1500 kg car can safely travel around a circular track of 80.0 m if the coefficient of static friction between the tire and road is 0.30.

## Universal Gravitation

Newton realized that the same gravity that keeps you on Earth also keeps the moon orbiting the Earth. This led to the idea of fields. A field is a region in space where one object can exert an influence on another object at a distance. A force is applied by one field acting on another similar field. The force of gravity involves the gravitational fields acting upon one another.

The field always points inwards towards the centre of mass. The gravitational force two objects experience are equal in magnitude but act in opposing directions. All masses have a gravitational filed, but only the gravitational fields of large masses are noticeable.

Changes in altitude and longitude affect the gravitational field strength on the surface of the Earth. Changes in the composition of the Earth's crust also affect this strength.

The gravitational field strength is the force acting on a 1 kg mass.

Let's go back to Newton and his apple…

Newton started with the idea that since the Earth, is pulling on the apple, the apple must be pulling on the Earth (Newton 3). If the apple is pulling on the Earth, that means that the object does not have to be huge to be a source of gravity. There is nothing special about the Earth compared to the apple that creates gravity. This means that any mass pulls on any other mass.

Using lots of Calculus and some impressive physics, Newton came up with two key concepts:

1) The force due to gravity between two objects is proportional to the two masses.
2) The force due to gravity is inversely proportional to the square of the distance between the two masses.

The following formula was eventually arrived at:

| | |
|---|---|
| $\vec{F}_g = \frac{Gm_1m_2}{\vec{d}^2}$ | Where Fg = force due to gravity (N)<br>G = Universal Gravitational Constant = 6.67 x $10^{-11}$ $Nm^2/kg^2$<br>m1 = mass of object one (kg)<br>m2 = mass of object two (kg)<br>d = distance between centres of the two objects (m) |

The Gravitational Constant, G, is a value that has been experimentally found. Physicists do not necessarily like formulas with constants, as it shows an incomplete understanding of a concept. Perhaps one day you will fix this embarrassing little problem for science.

Please note, G is not the same as g!

In practice, the weight of an object depends on its location with respect to one or more celestial bodies. Anything smaller doesn't exert much of a field.

Ex. You have a 200g pickles next to a 345 g sandwich on your plate. Their centres are 4.5 cm apart. Determine the force of gravity of the pickle pulling on the sandwich.

Ex. Io is one of Jupiter's moons. It is known to have 400 active volcanoes. One of the reasons for this is because of the gravitational pull of Jupiter on Io. At some time, the Sun, Io, and Jupiter are all lined up in a straight line so that both Sun and Jupiter were pulling Io in opposite directions. Using the data below, find the net force acting on Io at this time.

| Celestial Body | Distance from Io (m) | Mass (kg) |
|---|---|---|
| Io | - | $8.93 \times 10^{22}$ |
| Jupiter | $4.22 \times 10^{8}$ | $1.90 \times 10^{27}$ |
| Sun | $7.98 \times 10^{11}$ | $1.99 \times 10^{30}$ |

## Momentum

Momentum combines mass and velocity. It is similar to the idea of inertia – an object "has it". Inertia is different, though, because it is a concept and cannot be measured. Momentum can.

Momentum is calculated by multiplying the mass and velocity of an object.

| | |
|---|---|
| $\vec{p} = m\vec{v}$ | Where p = momentum (kgm/s)<br>m = mass (kg)<br>v = velocity (m/s) |

Ex. A 2352 kg boulder is rolling down a hill at 24 km/h. Calculate its momentum.

## Impulse

Impulse is a change in momentum. Since implies means the momentum has changed, the objects velocity must have changed. We will assume for all of our problems that it is the velocity, and not the mass of the object, that changes. Applying this thought to our momentum equation yields:

| | |
|---|---|
| $\Delta\vec{p} = m\Delta\vec{v} = \vec{J}$ | Where Δp = change in momentum (kgm/s)<br>m = mass of object (kg)<br>Δv = change in velocity (m/s)<br>J= impulse (change in momentum) |

Ex. Another boulder is rolling down the hill. This boulder has a paltry mass of 23 kg and is moving at 8.2 m/s. A dog tries to stop it, but only slows it down to 2.5 m/s. What impulse did the dog apply to the boulder?

The negative sign means the dog's impulse was acting in the negative direction. Momentum was taken away from the rock. If an impulse changes the velocity of an object, then it causes an acceleration. Where there is an acceleration, there is a force…

## Newton Two Redux

When Newton first wrote his Second Law of Motion, he didn't write it as F = ma. Instead, he said that force is proportional to the rate of change in momentum:

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$$

By substituting out formula for momentum in, one can arrive at the version of Newton's Second Law that we use. More importantly at this time, we can get:

$$\Delta\vec{p} = \vec{F}\Delta t$$

or

$$\vec{F}\Delta t = m\Delta\vec{v}$$

Each side of the equal sign in the equation above is equal to impulse. This is very versatile; you can use only the left side or only the right side to find the impulse, or you can use both sides to find one of the unknowns if you know the other three.

Ex. A rifle is firing a 9.00 g bullet so that it leaves the muzzle after 0.0300 s travelling at 200 m/s. Find the average force of the rifle acting on the bullet.

Impulse depends on two factors: force and time interval. To change an object's momentum, think of the following three situations:

Apply a medium force over a medium time interval.

Apply a big force over a small time interval to get the same impulse.

Apply a small force over a long time interval and still get the same impulse.

Use this to explain why you would rather land on a high jump mat instead of the ground.

Remember, it force that you feel, so you want that to be as small as possible.

Ex. After getting his Porsche out of the snowbank, 85 kg Mr. Birrell is travelling at 75 km/h when he runs into a chestnut tree.

a) If he had no airbag in his car and he came to rest against the steering wheel in 0.050s, determine how much force was exerted on his body.

b) If he did have an airbag that inflated and deflated correctly, bringing him to a rest over 0.78s, find how much force was exerted on his body.

c) What percentage of a) is b)?

## Conservation of Momentum in One Dimension

Because we can measure momentum, we could measure objects before and after they have a collision. A collision between objects results in an impulse. Any collision we look at must be in an isolated system. This means that no matter or energy is allowed to enter or leave the system – there are no external forces acting on the objects.

It was noticed in Newton's day that the total momentum of all objects before a collision equals the total momentum of all objects after. This is called the Law of Conservation of Momentum and is a fundamental law of physics. Any conservation law means that whatever you started with you still have at the end.

| Before Collision | After Collision |
|---|---|
| Momentum of object a = $p_a$ | Momentum of object a = $p_a'$ |
| Momentum of object b = $p_b$ | Momentum of object b = $p_b'$ |

The ' is pronounced "prime" and represents values after the collision.

When solving collision problems, one of two things will happen:

1. The two objects will bounce apart.
2. The two objects will stick together.

Ex. Object bounce apart

A 0.20kg green billiard ball moving at 9.2 m/s right hits a yellow striped ball at rest. If the green ball continues to move to the right at 2.5 m/s, find the velocity of the yellow ball.

Ex.  Objects stick together

A 95 kg linebacker tackles a 75 kg running back.  If the linebacker was moving towards the south goalpost at 8.4 m/s while the running back is moving at 5.3 m/s North, determine their final velocity.

While the above example showed two objects sticking together after the collision, it is also possible to have the reverse.  Sometimes, the objects start together and end up apart after the collision.  If this happens, just reverse the right and left sides of the equation in the above example.

# Unit V: Mechanical Energy

## Work

In physics, we have two definitions of work.

1) Work is a transfer of energy.

This means that energy changes forms or energy is transferred from one object to another object. When a transformation takes place, not all of the energy is used to produce useful work. Some is converted into heat or other types of energy. For now, just know that energy is the ability to do work.

2) Work happens when a force causes an object to move through a displacement.

If the force exerted on an object does not cause a displacement, no work is done. While causing a displacement, the force must also be in the same direction as the displacement.

The unit of work is the Nm. This is commonly called the Joule (J). Work is a scalar quantity, but comes from the product of two vectors. Positive work is done when the applied force and the displacement are in the same direction. Negative work is done when they are in opposite directions.

Ex. Holding a pizza in your hands, you walk 8 m to the other side of the room. Did you do any work?

Ex. Wanting to get stronger for next season, you go to the weight room to better yourself. If you lift 75 kg from the ground, over your head, and back down to the floor, have you done any work?

Ex. After getting his Porsche fixed, Mr. Birrell (again) gets stuck in a snowbank. He promises a passing student a passing grade if they do some work. Both people get behind the car and push it out of the snow. Did the student do work?

Ex. After getting tired of holding your 15 kg Physics binder, you put it on the floor. Did you do any work?

Both definitions of work lead to equations:

| | |
|---|---|
| $W = \Delta E$ | |
| $W = F\Delta d cos\theta$ | Where W = work (Joules) (J)<br>F = force (N)<br>Δd = displacement (m)<br>Θ = angle between the force and displacement vectors |

There is an angle in the second formula because it is possible for work to be done when the vectors of force and displacement are not parallel. The only time when no work is said to be done is when the forces are perfectly perpendicular to one another. The cosine θ calculates the component of the force in the direction of the displacement. So, if the force and displacement are perpendicular, the cosine of 90° is 0 and leads to no work.

Ex. Pulling your new insect overlord's sled at an angle of 42°.

Ex. You pull the sled horizontally with a force of 385 N for 48 m. Find the work done.

Ex. You pull the sled at an angle of 42° with a force of 385 N for 48 m until your insect overlord eats you. Determine the amount of work done before going to your painful death.

## Kinetic Energy

Energy is a scalar quantity measured in Joules. There are many types of energy; we will look at but three types.

Any object that is moving has kinetic energy. Like momentum, we can calculate how much kinetic energy an object possesses.

| | |
|---|---|
| $E_k = \frac{1}{2}mv^2$ | Where $E_k$ = kinetic energy (J)<br>m = mass (kg)<br>v = speed (m/s) |

We define any object as anything that has a mass. Also keep in mind that motion is relative. An object may have different amounts of kinetic energy depending upon its frame of reference. Usually, the surface of the Earth will be our reference point.

Ex. A distraught chicken is falling off a roof of a barn. How much kinetic energy does it have relative to its intestinal parasite?

How much kinetic energy does the 3.4 kg chicken have relative to someone standing on the ground if it is moving at 6.5 m/s?

Ex. Determine the speed of the 3.4kg chicken if it has 551J of kinetic energy.

Ex. Driving his 1500 kg Porsche down the road at 124 km/h, Mr. Birrell notices a school zone ahead. He hits the brakes and slows down to 37 km/h. If he slowed down over 39.6 m, find the average force applied by the brakes.

The concept that work is equal to the change in energy is called the Work-Energy Theorem. This means that doing work on an object changes the amount of mechanical energy it has. Mechanical energy is not just kinetic energy – the objects potential energy may change as well.

## Potential Energy

Potential energy is stored energy that is able to do work later.  There are many types.

Ex. Batteries-

Stretched Elastic-

Matter-

Holding a rock above your head-

## Gravitational Potential Energy

Gravitational potential energy is energy stored as a result of the position of an object relative to ground level or some other arbitrary reference called a base level.

| | |
|---|---|
| $E_p = mg\Delta h$ | Where $E_p$ = gravitational potential energy<br>m = mass of object being raised<br>g = acceleration due to gravity (m/s$^2$)<br>$\Delta$h = height (m) object is over base level |

Bizarrely, you can change the amount of potential energy an object has by moving it horizontally.

Ex.

Ex.  Determine the gravitational potential energy of a 3.4 kg dog 26.7 m above the ground.

Ex.  Determine the gravitational potential energy of the same dog at the same height using their owner's outstretched arms at 1.4 m above the ground as the base level.

Which is the correct answer for how much potential energy the dog has?

Ex. Determine the height of an 85 kg skydiver if they have $2.08 \times 10^7$ J of potential energy.

## Elastic Potential Energy

Anything that can have its shape changed and then return to its original shape can store elastic potential energy.

Robert Hooke, a man who did so many things for science, came up with Hooke's Law:

| | |
|---|---|
| $F_s = -kx$ | Where Fs = force (N)<br>k = spring constant (N/m)<br>x = amount of expansion (+) or compression (-) (m) |

The minus sign in the formula above means that the formula calculates the amount of force trying to restore the spring back to its original position.

Ex. A spring has a spring constant of 20.3 N/m. Find the restoring force when:

a) The spring is extended 12 cm.

b) The spring is compressed 21 cm from its starting point.

The above formula can be used to find a formula for elastic potential energy:

$$E_p = \frac{1}{2}kx^2$$

Ex. Find how much energy a composite hockey stick can store if it has a spring constant of 11000 N/m and it is flexed 0.12 m.

## Conservation of Energy

Often, it is useful to look at the total mechanical energy of a system. While there is no formal formula for total mechanical energy, it could look something like this:

$$E_m = E_k + E_p$$

Ex. A 2500 kg UFO is cruising at an altitude of 867 m while looking for a dog to kidnap. If it is travelling at a constant speed of 324 km/h, find the total mechanical energy of the UFO.

Recall that an isolated system means that no matter or energy can enter or leave. This means that you end with what you start with. No external forces are acting on the system.

If we have an isolated system, then energy is conserved. Total mechanical energy before will equal the total mechanical energy after.

$$E_m = E'_m$$

$$E_k + E_p = E'_k + E'_p$$

This is similar to the Law of Conservation of Momentum. In a twist of events that may shock you, this law is called the Law of Conservation of Energy. If one type of energy decreases, the other type will have to increase to balance everything out.

Ex. On a cold winter night, you find yourself atop a 35.8 m hill with your trusty GT Racer. If you and the GT have a combined mass of 81 kg, how fast will you be going at the bottom of the hill? Assume there is no friction.

Ex. In an attempt to discover the physics laws of our universe, the aforementioned UFO will be dropping a kidnapped dog from a height of 265 m. Find the velocity of the dog at the following heights above the ground, assuming no air resistance:

a) 265 m

b) 142 m

c) 0m

Ex. A toy car with mass of 212g is pushed by a student along a track so that it is moving at 12 m/s. It hits a spring (k = 52.8 N/m) at the end of the track, causing it to compress.

a) Determine how far the spring compressed to bring the car to a stop.

b) If the spring only compressed 50 cm in bringing the car to a stop, explain what happened.

## Power

Power is the rate at which work is done. It is a measure of how quickly energy is being used. Power is measured in J/s, which is also called the Watt (W).

| | |
|---|---|
| $P = \frac{W}{t} = \frac{\Delta E}{t}$ | Where P = power (Watts)<br>W = ΔE = work (Joules)<br>t = time (seconds) |

It is important to note that electricity is not power – electricity is energy.

Ex. If you leave a 150W bulb on for 8.5 h, determine how much electricity you have used.

Many of the questions we look at involved motors lifting something at a constant velocity. A higher power motor will be able to lift the object faster. When the object being lifted is moving at a constant velocity, we can use the following:

| | |
|---|---|
| $P = \frac{W}{t}$<br>$P = \frac{F\Delta d}{t}$<br>$P = Fv$ | Where P = power (W)<br>F = force (N)<br>v = average velocity (m/s) |

Ex. A 220 W motor is being used to lift plywood at a constant speed onto a roof. If one sheet of plywood has a mass of 34 kg, find the speed at which the plywood will be raised at.

## Efficiency

Efficiency can be measured in terms of either energy or power. It is a comparison of how much useful stuff comes out (output) compared to what was originally put in (input).

There is no such thing as a 100% efficient device – the output will always be less than the input. In any conversion, there is some waste energy that is turned into forms you don't necessarily want.

$$efficiency = \frac{Energy\ Output}{Energy\ Input} = \frac{E_{out}}{E_{in}}$$

$$efficiency = \frac{Power\ Output}{Power\ Input} = \frac{P_{out}}{P_{in}}$$

Ex. A 60 W incandescent bulb has an efficiency of only 2.0%. Find the amount of power that is turned into light.

Ex. A crane is lifting a 380 kg load at a constant velocity of 3.2 m/s. Determine its efficiency if the motor on the crane is rated at 22 000 W.

How many horsepower is the engine in the above example?

# Unit VI: Electricity

## Coulomb's Law

Remember electrostatics:

Like charges repel.

Opposite charges attract.

Charge is measured in Coulombs (C). It is named in honour of Charles Augustin de Coulomb.

Common subatomic particles can have a charge, as summarized in this table:

| **Particle** | **Charge** |
|---|---|
| Electron ($e^-$) | $-1.60 \times 10^{-19}$ C |
| Proton ($p^+$) | $+1.60 \times 10^{-19}$ C |
| Neutron ($n^0$) | 0 C |

A charge of $1.60 \times 10^{-19}$ C is called an elementary charge and has a symbol of "e". Do not confuse this with the symbol for electron, as there is no negative sign on the symbol.

One C is equivalent to the charge on $6.25 \times 10^{18}$ particles.

Ex. What is the charge on 12 protons?

| | |
|---|---|
| $q = ne$ | Where q = charge (C)<br>n= number of elementary particles<br>e = $1.60 \times 10^{-19}$ C |

Coulomb found that the force between charges is dependent on the charge of the objects and their distance apart.

| | |
|---|---|
| $\lvert F_e \rvert = \dfrac{kq_1q_2}{r^2}$ | Where Fe = force (N)<br>q = charge (C)<br>r = distance between the charges (m)<br>k = $8.99 \times 10^9$ Nm$^2$/C$^2$ |

Coulomb's Law will just give us the magnitude of the electric force. We will use information about the charge on the objects to determine the direction. The positive and negative values from the formula above do not come from direction, but from positive and negative charges.

Ex.  A comb with -2.0 μC of charge is 0.15 m to the left from a hair with 3.0 μC of charge.  Determine the force the hair exerts on the comb.

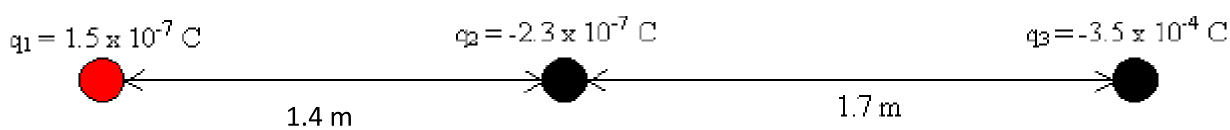
In the question above, the charges were in micro coulombs.  Charges that are seen in day to day life tend to be about 1 μC.  Things like a lightning bolt may have a charge of 1 or 2 C.

You may also notice that Coulomb's Law is very similar to Newton's Law of Universal Gravitational.  What is one significant difference between the two?

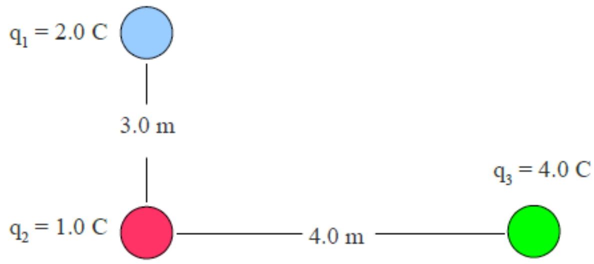
What does this say about the strength of the gravitational force versus that of the electrostatic force?

Another important difference is that the gravitational force can only cause an attraction, but the electrostatic force can cause attraction or repulsion.

Ex.  The following three charges are arranged as shown.  Determine the net force acting on the far right.

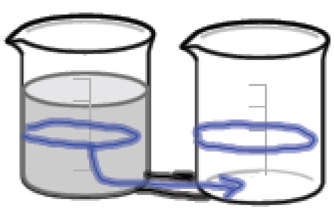


Ex. Three charges are arranged in a right angle triangle as the diagram shows. Determine the force on $q_2$.

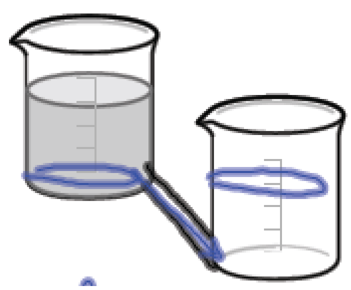



## Flow of Charge

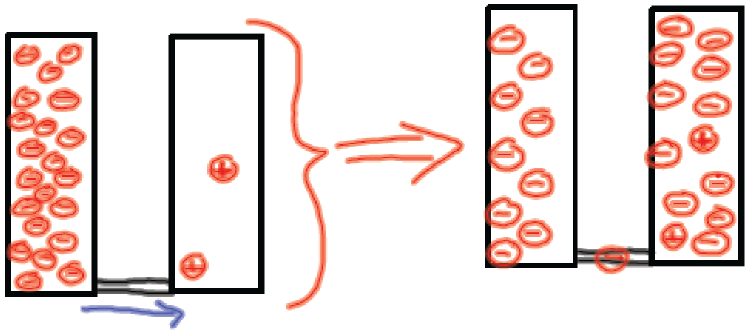
Water Analogy:



Water flows because the potential energy differences that exist between the tops of the water.



Charge flows due to a potential difference in charge between the two charged objects. The charge will flow until it is in equilibrium.



When a conductor is connected between both terminals of a battery, it forms an electric circuit. The battery in the circuit will cause charge to flow from one terminal to another. This is called electric current.

A better definition of current is that it is the amount of charge that passes a given point in a certain amount of time.

| | |
|---|---|
| $I_{avg} = \frac{q}{\Delta t}$ | Where $I_{avg}$ = current in amperes (A)<br>q = charge in coulombs (C)<br>Δt = time in seconds (s) |

Ex. A current of 1.42 A flows through a wire connecting the terminals of a battery. After 4.00 minutes, how much charge has passed through the circuit?

Remember, it is the free electrons in a conductor that can move around. And while we talk about the electrons "flowing", they are really only wiggling back and forth.

## Voltage

Voltage is sometimes referred to as the electric potential difference, the electric potential, or potential difference. The electric potential difference is the reason that current flows – without it, nothing would happen. It is the force that pushes the electrons. Voltage is the electric potential energy per unit charge. It is how much work is needed per Coulomb of charge. If something has more charge, it needs more work to move it.

The Volt (V) is the derived until named in honour of Alessandro Volta. One V is equivalent to a J/C.

| | |
|---|---|
| $V = \frac{W}{q} = \frac{\Delta E}{q}$ | Where V = voltage (V)<br>W = ΔE = electric potential energy (J)<br>q = charge (C) |

Ex. How much voltage is needed to do 2500 J of work moving $3.5 \times 10^{-3}$ C?

## Resistance

The amount of current flowing in a circuit depends partly upon the voltage. If you increase the voltage, you increase the "pumping power" that is moving current through the circuit. Current flow also depends on how the material of the conductor resists the motion of electrons.

Often, a water pump is used as an analogy for electricity.

The pump is like voltage. It is the pumping power that is shoving electrons through the wire.

The hose diameter is like the wire's resistance. A thin hose lets only a little water through, just like a poor conductor lets only a little current through.

The water is like current. A little water pouring out is like a little current running through a wire.

Resistance depends upon four factors:

| Property | Relationship | Example |
|---|---|---|
| Resistivity of material | Different materials used as conductors result in different resistances. This is compared by assigning a resistivity (ρ) to materials. | Gold has a low resistivity because it is such a good conductor. |
| Cross section area of wire | Larger area results in low resistances. | A large diameter wire is a wider opening for electrons to move through, just like a wide doorway can allow more people to walk easily through. |
| Length of wire | Shorter lengths of wire result in low resistances. | Moving current through a long piece of wire is like forcing the electrons to run a marathon race. A lot of energy is used up just trying to get to the end. |
| Temperature | Higher temperatures result in higher resistances. | When conductors are colder, their atoms move less. This makes it easier for electrons to "move through". |

We will not take into account the temperature coefficient (α) in our calculations.

| | |
|---|---|
| $R = \frac{\rho l}{A}$ | Where R = resistance (ohms = Ω)<br>ρ = resistivity (Ωm)<br>l = length<br>A = area ($m^2$) |

Ex. Wire gauge is a method used to compare different cross sectional area of wire; the higher the gauge, the smaller the area. 14-gauge wire has a diameter of 1.628 mm. If aluminum has a resistivity of $2.82 \times 10^{-8}$ Ωm, determine the resistance of a 12.5 m long piece of aluminum wire.

## Ohm's Law

Georg Simon Ohm made a discovery about certain conductors. If a conductor's resistance stays constant even when different voltages are applied to it, the conductor is said to obey Ohm's Law. For our course, all conductors will obey Ohm's Law.

| | |
|---|---|
| V = IR | Where V = voltage (V)<br>I = current (A)<br>R = resistance (Ω) |

Ex. A 30Ω resistor is hooked up to a 12 V battery. How much current flows?

## Transfer of Energy

Like other forms of energy, electricity can change forms. Combining equations from this chapter and from work and energy, we get the following:

| | |
|---|---|
| $P = \frac{qV}{t}$ | $P = VI$ |
| $P = \frac{V^2}{R}$ | $P = I^2R$ |

These will be the main formulas we will use when calculating power in an electrical circuit.

Ex. A household appliance draws a 0.50 A current. Determine the power rating of this appliance.

Ex. A portable fridge in the back of a van has a power rating of 5.0 W. Since it is plugged into the van's electrical system, it runs off of 12 V. Determine the resistance of the fridge.

We can also combine a number of equations to get relationship for electrical energy.

| | | |
|---|---|---|
| $E=P\Delta t$ | $E=VI\Delta t$ | $E=I^2R\Delta t$ |

Ex. A heater has a resistance of 10.0 Ω. It operates on 120.0 V.

What is the current through the resistance?

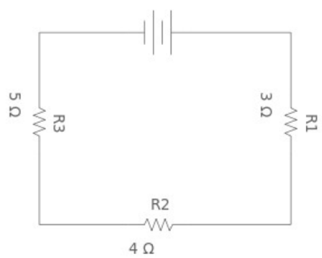
What thermal energy in joules is supplied by the heater in 10.0 s?

## Circuits

Any path along which electrons can flow is a circuit. For a continuous flow of electrons, there must be a complete circuit with no gaps. Circuits can be broken down into two main categories: series and parallel.

In any circuit, going through a battery increases voltage and going across a resistor decreased voltage (a voltage drop). As you go around any single pathway, they total voltage drops across all resistors must equal the voltage from the battery.

## Series Circuits




In a series circuit, there is only one path for the electrons to flow. The current is constant everywhere in a series circuit. If you break the circuit at any point, the entire circuit will stop working.

The current has to flow through all the resistors, so we increase the resistance of the circuit as we add more resistors. Since the total drops in voltage must equal the voltage from the battery along a single path, we can say:

$$V_T = V_1 + V_2 + V_3 + \cdots$$

Since V = IR, we can also say:

$$IR_T = IR_1 + IR_2 + IR_3 + \cdots$$

Because the current is the same anywhere in a series circuit, we can divide both side by I and get:

$$R_T = R_1 + R_2 + R_3 + \cdots$$

The 'T' subscript refers to the total voltages and resistances of the entire circuit. This means that if we take all the resistances and add them up, we get a single equivalent resistance. At that point, you can use Ohm's Law to find the actual current going through the circuit.

Ex. Determine the voltage drops and current flowing through each part of a circuit containing a 9 V battery, a 3 Ω resistor, a 4 Ω resistor, and a 5 Ω resistor.

| | V | I | R |
|---|---|---|---|
| $R_1$ | | | |
| $R_2$ | | | |
| $R_3$ | | | |
| $R_T$ | | | |

Find equivalent resistance:

| | V | I | R |
|---|---|---|---|
| $R_1$ | | | 3 |
| $R_2$ | | | 4 |
| $R_3$ | | | 5 |
| $R_T$ | 9 | | |

Use Ohm's Law to find current:

| | V | I | R |
|---|---|---|---|
| $R_1$ | | | 3 |
| $R_2$ | | | 4 |
| $R_3$ | | | 5 |
| $R_T$ | 9 | | 12 |

The current is the same everywhere, so we can use that value of I for all the resistors.

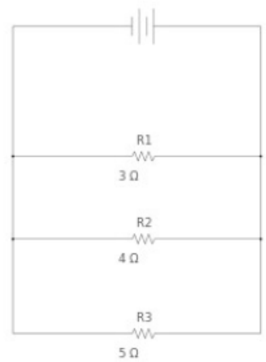
| | V | I | R |
|---|---|---|---|
| $R_1$ | | | 3 |
| $R_2$ | | | 4 |
| $R_3$ | | | 5 |
| $R_T$ | 9 | 0.75 | 12 |

Use Ohm's Law to find the voltage drop of each resistor:

| | V | I | R |
|---|---|---|---|
| $R_1$ | | 0.75 | 3 |
| $R_2$ | | 0.75 | 4 |
| $R_3$ | | 0.75 | 5 |
| $R_T$ | 9 | 0.75 | 12 |

You can check your work by making sure that the sum of the voltage drops across the three resistors equals the same voltage as your power source.

## Parallel Circuits




In a parallel circuit, there is more than one pathway for the electron flow between terminals. Since each resistor is in direct connection to both terminals of the battery, each resistor has the same full voltage from the battery in a parallel circuit.

As long as it isn't on the main branch, a break anywhere in the circuit does not affect the other resistors if they still have a path to the battery.

Because charge is conserved, the current flowing into a junction must equal the current flowing out. Similar to what we did with series circuits, we get the following:

$$I_T = I_1 + I_2 + I_3 + \cdots$$

$$\frac{V}{R_T} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} + \cdots$$

Because in a parallel circuit the voltage is the same among all the resistors, we can divide out V:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

Ex. Determine the voltage drops and current flowing through each part of a circuit containing a 9 V battery, a 3 Ω resistor, a 4 Ω resistor, and a 5 Ω resistor if the resistors are all wired in parallel.

| | V | I | R |
|---|---|---|---|
| $R_1$ | | | |
| $R_2$ | | | |
| $R_3$ | | | |
| $R_T$ | | | |

Scrunch the parallel resistors down to one equivalent resistor:

Calculate the current using the equivalent circuit:

| | V | I | R |
|---|---|---|---|
| $R_1$ | | | 3 |
| $R_2$ | | | 4 |
| $R_3$ | | | 5 |
| $R_T$ | 9 | | |

The voltage is the same everywhere so we can unscrunch the circuit and put in the voltage for each resistor:

| | V | I | R |
|---|---|---|---|
| $R_1$ | | | 3 |
| $R_2$ | | | 4 |
| $R_3$ | | | 5 |
| $R_T$ | 9 | 7.05 | 1.276595745 |

Now we can solve for the missing currents in each of the individual branches of each resistor:

| | V | I | R |
|---|---|---|---|
| $R_1$ | 9 | | 3 |
| $R_2$ | 9 | | 4 |
| $R_3$ | 9 | | 5 |
| $R_T$ | 9 | 7.05 | 1.276595745 |

To check your work, add all of the currents going through the separate branches. It should add up to the current on the main branch.

## Kirchoff's Laws

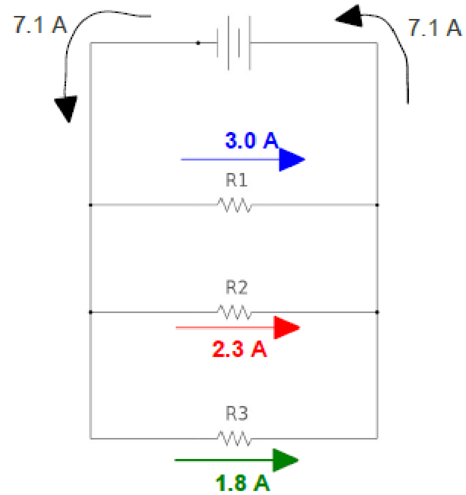
Sometimes, we encounter a circuit that is too complicated for simple analysis. This may be because the circuit is a mix of series and parallel, or has more than one power source. G. R. Kirchoff invented some rules to deal with these cases. The rules boil down to convenient applications of the laws of conservation of charge and energy.

In the examples above, you were already sort of using his rules, unbeknownst to you.

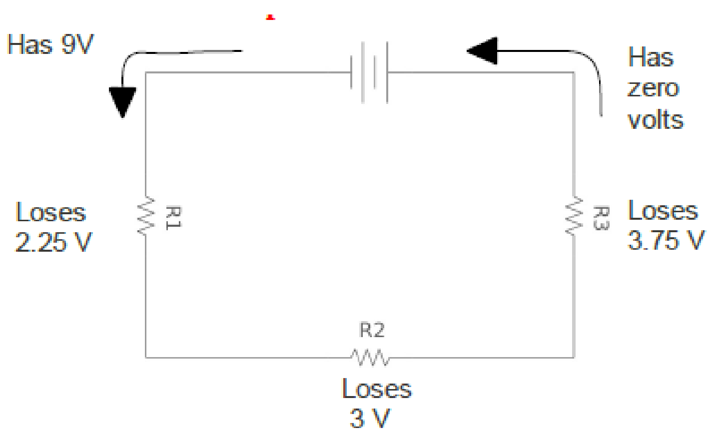
The First Rule : The Junction Rule:

"At any junction point, the sum of all currents entering the junction must equal the sum of all currents leaving the junction."

This means that when current reaches the branches in a parallel circuit, it will split up and take different routers. When the branches come back together, the currents will add back together too.




The Second Rule: The Loop Rule




"The algebraic sum of the changes in potential around any closed path of a circuit must be zero."

This rule is based on the conservation of energy. This just means that when you look at resistors in series, the drop in voltage across all of them will be equal to whatever the source is.

These two rules become important when you need to analyze combination circuits.

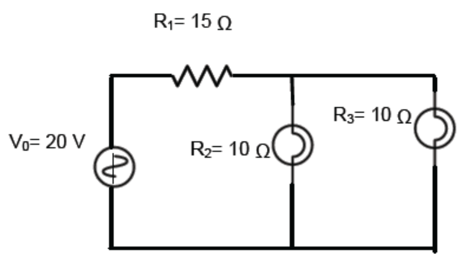
## Combination Circuits

Combination circuits are more realistic because they are circuits with series and parallel parts together. You'll recognize them when you see at least a couple of resistors are in parallel, but after you scrunch them, you end up with a series circuit for the other resistors.

To solve circuits that are a combination of series and parallel, flip back and forth through Kirchoff's Laws as appropriate. Use Ohm's Law any time you know two things at one point.

As a general rule, try to scrunch the parallel parts first. Once that is done, scrunch the series parts. Skipping steps leads to errors. You might have to scrunch resistors in series within a parallel circuit first.

Ex. Solve the following circuit.




| | V | I | R |
|---|---|---|---|
| $R_1$ | | | |
| $R_2$ | | | |
| $R_3$ | | | |
| $R_T$ | | | |